

Trapped-ion qutrit spin molecule quantum computer

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We present a qutrit quantum computer design using trapped ions in the presence of a magnetic field gradient. The magnetic field gradient induces a "spin-spin" type coupling, similar to the J-coupling observed in molecules, between the qutrits which allows conditional quantum logic to take place. We describe in some detail, how one can execute specific one and two qutrit quantum gates, required for universal qutrit quantum computing.

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Quantum information and quantum computing [1], have made huge advances both theoretically and experimentally in recent times. There are many different proposals [2, 3, 4, 5, 6, 7, 8, 9, 10, 11], for the physical implementation of a quantum computer, all of which are specified by the physics of their qubit systems and the nature of the interactions between qubits. The latter is necessary for the execution of conditional qubit logic, a requirement for universal qubit quantum computation. The qubit is of course the obvious unit of a quantum computer given our classical computer historical dependance on binary logic. Quantum algorithms and protocols for quantum communication and cryptography have been studied extensively with the qubit as the information storage and transport medium. Recently d -level quantum systems, or qudits, have started to be considered seriously in terms of generalizing and improving qubit-based quantum algorithms and protocols. Interesting results have emerged in the qutrit ($d = 3$) case. It has been shown that quantum cryptography protocols are more robust against eavesdropping attacks when qutrits are used [12, 13, 14]. Quantum bit commitment and coin-flipping protocols are more secure with entangled qutrits than with qubits [15]. Indeed, it is expected that qutrit-based quantum information processing will be more powerful than other qudit implementations since they optimize the Hilbert space dimensionality [17]. More speculative is the possibility of a higher error-tolerance for fault-tolerant qutrit quantum computation [18]. However, there have not been many qudit-based quantum computer proposals. As far as we are aware there has been only one qutrit quantum computer proposal using trapped ions which generalises the original Cirac-Zoller design [19].

Here we describe a modification to previous work [20, 21, 22], on ion trap quantum computers where now qutrits store the quantum information. An axial magnetic field gradient is applied across an ion chain that allows the three hyperfine Zeeman energy levels of each

ion, forming the qutrit, to be individually frequency addressed. It also introduces an inter-qutrit coupling that facilitates conditional quantum logic between qutrits. Previously the operation of an ion trap quantum computer in the presence of a magnetic field gradient has been discussed with qubits as the unit of quantum information. In [20, 21], it is shown that all quantum gate operations, normally requiring optical irradiation, can be implemented using long wavelength radiation due to the effects of the magnetic field gradient and trapping potential. The gradient also introduces a term in the Hamiltonian that is analogous to the spin-spin coupling observed between nuclei in molecules in NMR. This coupling can be used to perform quantum logic. This idea is investigated further for ions in a linear array of microtraps [22]. In the only other qutrit ion trap quantum computer proposal [19], the quantized collective vibrational motion of the linearly trapped ions is used as a quantum bus to preform quantum logic.

In this work we consider N ions in a linear ion trap in the presence of a magnetic field gradient. Three unequally spaced hyperfine Zeeman levels serve as our qutrit (Fig 1). We will refer throughout to the $^{171}\text{Yb}^+$ ion as an example qutrit, particularly the $F = 1$ hyperfine Zeeman levels shown in Fig. 2. Our computational basis is now $\{|0\rangle, |1\rangle, |2\rangle\}$ written as $|0\rangle = (0, 0, 1)^\top$, $|1\rangle = (0, 1, 0)^\top$ and $|2\rangle = (1, 0, 0)^\top$. We denote by $\omega_{01}^{(n)}$ and $\omega_{12}^{(n)}$ the frequencies resonant with the $|0\rangle \leftrightarrow |1\rangle$ and $|1\rangle \leftrightarrow |2\rangle$ transitions for the n th ion and let $\Delta_n = \omega_{01}^{(n)} + \omega_{12}^{(n)}$ and $\delta_n = \omega_{01}^{(n)} - \omega_{12}^{(n)}$. For each ion these frequencies, due to the spatial dependence of the magnetic field, are a function of the ions' positions. The internal electronic Hamiltonian describing the spin degrees of freedom of a single ion is given by $H_{sp,n} = \frac{1}{2}\hbar Z_n$ with Z_n expressed in the above computational basis as

$$Z_n = \begin{pmatrix} \Delta_n & 0 & 0 \\ 0 & \delta_n & 0 \\ 0 & 0 & -\Delta_n \end{pmatrix}. \quad (1)$$

As in [20, 21], the ions sit in an effectively 1d harmonic oscillator potential along the trap axis and feel their mutual Coulomb repulsion. Expanding this potential around

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their equilibrium positions allows their vibrational motion to be treated collectively and the Hamiltonian for the motional degrees of freedom of the ions in normal coordinates is

$$H_{vib} = \frac{1}{2m} \sum_n P_{Q,n}^2 + \frac{m}{2} \sum_n \nu_n^2 Q_n^2. \quad (2)$$

The local and normal coordinates are related by $q = DQ$, with D the unitary transformation that diagonalises the Hessian, A , of the ions potential evaluated at the equilibrium positions of the ions, $z_{0,n}$ [See refs [21, 23]].

The magnetic field gradient, $\frac{dB}{dz} \equiv b$, means the ions feel a spatially varying magnetic field, $B(z) = B_0 + b \cdot z$. This introduces a new term into the spin part of the Hamiltonian,

$$H_{el,n} = \frac{1}{2} \hbar Z_n|_{z_{0,n}} + \frac{\hbar}{2} \frac{dZ_n}{dz} \Big|_{z_{0,n}} q_n. \quad (3)$$

Setting $M_n = dZ_n/dz|_{z_{0,n}}$ for easier notation, as long as $< \frac{1}{2} \hbar M_n q_n >$ is much smaller than the ground state energy of the collective vibrational motion, then this new term will have a negligible effect on the normal modes and can be treated as a perturbation. This places a limit on the size of the magnetic field gradient but is no more stringent than other constraints discussed later. The resultant Hamiltonian is

$$\begin{aligned} H &= \frac{\hbar}{2} \sum_n Z_n + \frac{\hbar}{2} \sum_n \hat{M}_n \sum_l D_{ln} Q_l \\ &\quad + \frac{1}{2m} \sum_n P_{Q,n}^2 + \frac{m}{2} \sum_n \nu_n^2 Q_n^2, \\ &= \frac{\hbar}{2} \sum_n Z_n + \frac{1}{2m} \sum_l P_{Q,n}^2 \\ &\quad + \frac{m}{2} \sum_l \nu_l^2 \left(Q_l + \frac{\hbar}{2m\nu_l^2} \sum_n M_n D_{ln} \right)^2 \\ &\quad - \frac{m}{2} \sum_l \nu_l^2 \left(\frac{\hbar}{m\nu_l^2} \sum_n M_n D_{ln} \right)^2. \end{aligned} \quad (4)$$

We move to a rotating frame where the spin and motional degrees of freedom are decoupled via the unitary transformation $U = \exp \left[i \left(\frac{\hbar}{2m\nu_l^2} \sum_n M_n D_{ln} \right) P_{Q,l} \right]$, yielding $\tilde{H} = U H U^\dagger$, with

$$\tilde{H} = \frac{\hbar}{2} \sum_n Z_n + \sum_n \hbar \nu_n a_n^\dagger a_n - H_{MM}, \quad (5)$$

where $H_{MM} = \frac{m}{2} \sum_l \nu_l^2 \left(\frac{\hbar}{m\nu_l^2} \sum_n M_n D_{ln} \right)^2$, and the position and momentum operators have been expressed in terms of creation and annihilation operators in the usual way. H_{MM} can be expressed as $H_{MM} = \frac{\hbar}{2} \sum_{n < m} J_{nm} M_n M_m$ where

$$J_{nm} = \frac{\hbar}{2m} \sum_l \frac{1}{\nu_l^2} D_{ln} D_{lm}. \quad (6)$$

The Hamiltonian \tilde{H} in (5), describes N individually addressable qutrits coupled through H_{MM} , a “spin-spin” type interaction which we will presently show can be used to perform conditional quantum logic between qutrits. There are no extra experimental requirements compared to the setup proposed in [20, 21].

It is necessary to demonstrate how single qutrit operations and conditional logic, the most basic ingredients of a universal quantum computation, can be implemented in this modified design.

In order to perform single qutrit operations, we use the fact that the two transitions $|0\rangle \leftrightarrow |1\rangle$, and $|1\rangle \leftrightarrow |2\rangle$, have different resonant frequencies allowing the operations U_{01} , and U_{12} ,

$$\begin{aligned} U_{12}(\theta, \phi) &= \begin{pmatrix} \cos \frac{\theta}{2} & i e^{i\phi} \sin \frac{\theta}{2} & 0 \\ i e^{-i\phi} \sin \frac{\theta}{2} & \cos \frac{\theta}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ U_{01}(\theta, \phi) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \frac{\theta}{2} & i e^{i\phi} \sin \frac{\theta}{2} \\ 0 & i e^{-i\phi} \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix}, \end{aligned} \quad (7)$$

to be carried out. Since the $|0\rangle \leftrightarrow |2\rangle$ transition is forbidden, any rotation between these two states requires a π -pulse between $|0\rangle$ and $|1\rangle$, $U_{01}(\frac{\pi}{2}, 0)$, the required gate on $|1\rangle \leftrightarrow |2\rangle$, $U_{12}(\theta, \phi)$, followed by another π -pulse on $|0\rangle \leftrightarrow |1\rangle$. Also required is the unitary differential phase operation,

$$U_D = \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & e^{-i(\sigma+\rho)} \end{pmatrix}, \quad (8)$$

where this gate is a composition of “ σ_z ” operations on the $|0\rangle \leftrightarrow |2\rangle$ and $|0\rangle \leftrightarrow |1\rangle$ transitions that are themselves compositions of U_{12} and U_{01} operations. In particular, $U_D = (Z_{02})_\rho (Z_{01})_\sigma$ with $(Z_{ij})_\rho = H_{ij} U_{ij}(\rho, 0) H_{ij}^\dagger$ and $H_{ij} = U_{ij}(\frac{\pi}{4}, \frac{\pi}{2})$ is the Hadamard gate. Thus the unitary operations given in (7) allow us to generate any operation in $SU(3)$ [24].

For gates between more than one qutrit, we propose to use the last part of the Hamiltonian \tilde{H} in (5). We now consider in more detail the hyperfine Zeeman levels for the qutrits. For intermediate magnetic field strengths B , such that $g_J \mu_B B \approx A$, where g_J is the Landé g-factor, μ_B , is the Bohr magneton and A , is the hyperfine constant, the energy levels are described by the Rabi-Breit formula [25]. In this case $d\Delta_n/dz = g_J \mu_B b/\hbar$, and $d\delta_n/dz \approx g_J \mu_B b/\sqrt{2}\hbar$, and where b is the constant gradient of the magnetic field $B(z) = B_0 + bz$. We absorb the $g_J \mu_B b/\hbar$ factor into the definition of J_{nm} and write $M = dZ/dz|_{z_0}$ in the matrix form

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (9)$$

with now $J_{nm} = \frac{(g_J \mu_B b)^2}{2\hbar} (A^{-1})_{nm}$ [22]. Expressing the operator M in (9) in terms of the generators of $SU(3)$

we have

$$M = a_0 \mathbb{I} + a_3 \lambda_3 + a_8 \lambda_8, \quad (10)$$

where

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \quad (11)$$

with $a_0 = \frac{1}{3\sqrt{2}}$, $a_3 = \frac{1}{2\sqrt{2}}(\sqrt{2} - 1)$, and $a_8 = \frac{1}{2\sqrt{6}}(1 + 3\sqrt{2})$. H_{MM} is thus a two-body N-qutrit Hamiltonian as defined in [26], and so, as arbitrary local unitaries are possible, universal quantum computation can be performed. More specifically, it has been pointed out in [19] that a generalized XOR gate between qutrits,

$$\text{XOR}_{mn}|j\rangle_m|k\rangle_n = |j\rangle_m|j \oplus k\rangle, \quad (12)$$

where the “ \oplus ” operation now indicates addition modulo 3, can be decomposed into three operations,

$$\text{XOR}_{mn} = F_m P_{mn} F_m^{-1}. \quad (13)$$

The generalized Fourier transform, F , for one qutrit is defined by

$$F|j\rangle = \frac{1}{\sqrt{3}} \sum_{l=0}^2 e^{2\pi i j l / 3} |l\rangle, \quad (14)$$

where $j = 0, 1, 2$.

The phase gate for qubits, $P_{qub}|j\rangle_m|k\rangle_n = \exp(i\pi j k)|j\rangle_m|k\rangle_n$, is sandwiched by two Hadamard gates on the target qubit to give the controlled-NOT or XOR operation. The generalization of this gate to qutrits is P_{mn} . It is completely specified by

$$P_{mn}|j\rangle_m|k\rangle_n = \exp(2i\pi j k / 3)|j\rangle_m|k\rangle_n. \quad (15)$$

We can generate this gate between 2 qutrits using a combination of single qutrit rotations and evolution under the two qutrit Hamiltonian $H_{12} = 2\pi J M_1 \otimes M_2$. One pulse sequence for P_{mn} is

$$\begin{aligned} & \left(Z_{01}^{(1)} \right)_{\alpha_1} \left(Z_{12}^{(1)} \right)_{\alpha_2} \left(Z_{01}^{(2)} \right)_{\alpha_3} \left(Z_{12}^{(2)} \right)_{\alpha_4} \\ & \cdots (MM)_{\alpha_5} \left(X_{01}^{(1)} \right) (MM)_{\alpha_6} \left(X_{01}^{(1)} \right) \\ & \cdots \left(X_{12}^{(1)} \right) (MM)_{\alpha_7} \left(X_{12}^{(1)} \right) \left(X_{12}^{(2)} \right) \\ & \cdots (MM)_{\alpha_8} \left(X_{12}^{(1)} \right) (MM)_{\alpha_9} \left(X_{12}^{(1)} \right) \left(X_{12}^{(2)} \right) \end{aligned} \quad (16)$$

where $(X_{kl}^{(n)})$ represents a π -pulse on the $|k\rangle \leftrightarrow |l\rangle$ transition of the n th ion i.e. $(X_{kl}^{(n)}) = U_{kl}(\frac{\pi}{2}, 0)$ and $(MM)_\theta$ represents a period of evolution under the Hamiltonian H_{12} for a time t such that $2\pi J t = \theta$. While this is not a unique decomposition, the number of periods of evolution under H_{MM} is minimal. The angles $\{\alpha_i\}_{i=1}^9$ have been numerically determined and are shown in Table I.

Refocussing techniques [1] developed for NMR quantum computing are necessary here given the “always-on” nature of H_{MM} . For qubits the interaction is $H_{SS} = 2\pi J \sigma_z^{(1)} \sigma_z^{(2)}$. The relations $\sigma_a \sigma_z \sigma_a = -\sigma_z$ for $a = x, y$, where $\{\sigma_i\}_{i=x,y,z}$ are the Pauli operators, are used to reverse the evolution under H_{SS}

$$e^{itH_{SS}} = e^{-i\frac{\pi}{2}\sigma_x^{(1)}} e^{-itH_{SS}} e^{-i\frac{\pi}{2}\sigma_x^{(1)}}. \quad (17)$$

Essentially the diagonal elements of σ_z are permuted by the two π -pulses. Combined with another period of evolution under H_{SS} , the trace-less property of σ_z is exploited so that nothing happens. In fact, all qubits coupled to the first qubit are refocussed by this.

For qutrits the de-coupling procedure is basically the same. The spin-spin term $H_{MM} = 2\pi J M_1 M_2$ is composed of nine terms,

$$M_1 M_2 = a_0 \mathbb{I} \otimes M + (a_3 \lambda_3 + a_8 \lambda_8) \otimes M \quad (18)$$

By applying pulses which simultaneously permute the entries of λ_3 , and λ_8 , over three different permutations, in a manner similar to the qubit case, the combined evolution under the second term in (18) is removed. The sequence to refocus the MM evolution over an arbitrary duration T , requires one to subdivide the duration into three equal-duration sub-evolutions $(MM)_\theta$, $(2\pi J T / 3 = \theta)$. One can then obtain,

$$e^{i\phi} U_1 U_2 U_3 R_1^{(2)} R_2^{(2)} = \mathbb{I} \quad (19)$$

where

$$\begin{aligned} U_1 &= (MM)_\theta \\ U_2 &= X_{01}^{(1)} X_{12}^{(1)} (MM)_\theta X_{12}^{(1)} X_{01}^{(1)} \\ U_3 &= X_{12}^{(1)} X_{01}^{(1)} (MM)_\theta X_{01}^{(1)} X_{12}^{(1)} \end{aligned} \quad (20)$$

and

$$\begin{aligned} R_1^{(2)} &= \exp(3a_0 a_3 \theta) \\ R_2^{(2)} &= \exp(3a_0 a_8 \theta). \end{aligned} \quad (21)$$

The global phase factor, $\phi = 3a_0^2 \theta$, is irrelevant while the two single-qutrit pulses on the second qutrit are due to the first term in (18) and can be easily reversed. The three periods of evolution required are expected since the matrices are 3-dimensional.

Readout of the final state of the qutrit register takes two steps. The entire ion chain is illuminated with optical radiation and the observed fluorescence spatially resolved. The radiation frequency is chosen so that if the qutrits are projected onto $|2\rangle$ this is then detected. A π -pulse is then applied on the $|1\rangle \leftrightarrow |2\rangle$ transition of each ion and the ion string illuminated again. Fluorescence now indicates projection onto $|1\rangle$ while its absence means the qutrit is in $|0\rangle$.

We now describe explicit example using the $^{171}\text{Yb}^+$ ion. The hyperfine constant for $^{171}\text{Yb}^+$ is $A =$

2π 12.6GHz. In a magnetic field its levels are split as shown in Fig. 2. In fields of around 0.45T, the Rabi-Breit region, the good quantum numbers are F and M_F and our logical states are $|0\rangle = |6S_{\frac{1}{2}} F=1, M_F=-1\rangle$, $|1\rangle = |6S_{\frac{1}{2}} F=1, M_F=0\rangle$, and $|2\rangle = |6S_{\frac{1}{2}} F=1, M_F=1\rangle$. The transition frequencies ω_{01} , and ω_{12} , are about 3.7GHz and 8.9GHz, respectively while the differences between the resonance frequencies of two neighbouring ions in a trap of frequency 2π 200kHz and a magnetic field gradient of $b = 100\text{T/m}$ are $\delta\omega_{01} \approx 11\text{ MHz}$, and $\delta\omega_{12} \approx 2\text{ MHz}$. The operator M_n in (9) has an element which is approximately $1/\sqrt{2}$. This approximation introduces a constraint on the size of the magnetic field gradient if we require M_n to be constant over the ion chain within an accepted error ϵ_M . Numerical calculations in [27, 28], give the minimum distance between ions in a chain as $\Delta z_{\min}(N) = 2.018\gamma N^{0.559}$ where $\gamma = (q^2/4\pi\epsilon_0 m\nu_1^2)^{\frac{1}{3}}$. As a conservative estimate, let us say that the ions are equally spaced at $\delta z = 1.5\Delta z_{\min}(N)$. This implies that $b < 0.03N^{-0.441}\nu_1^{\frac{2}{3}}$, limiting the size of the magnetic field gradient. On the other hand we need to be able to frequency discriminate between qutrit transitions on neighbouring ions, and be far enough away that no vibrational motion will be excited i.e. $(d\omega_{01}/dz)\delta z \leq 2\nu_N + \nu_1$. Using the numerical result [20] that $\nu_N = (2.7 + 0.5N)\nu_1$, this imposes a minimum size on the magnetic field gradient of $b \geq 1.5 \times 10^{-9}\nu_1^{\frac{2}{3}}(3.2N^{0.559} + 0.5N^{1.559})$. For

an axial trap frequency of $\nu_1 = 2\pi$ 200kHz containing 10 ions and $\epsilon_M = 0.01$ the magnetic field gradient is limited by $30\text{T/m} \leq b \leq 200\text{T/m}$. These limits also determine the maximum number of ions we can place in the trap, assuming other considerations related to the ratio of the axial and radial trapping frequencies are satisfied [23]. For an axial trap frequency of $\nu_1 = 2\pi$ 200kHz, the maximum number of ions is about 30 satisfying the above limits when the magnetic field gradient is 120T/m. The expression for the J-couplings in (6) is the same as that in the qubit case. For 10^{171}Yb^+ ions at a trap frequency of 2π 200kHz and magnetic field gradient 120T/m gives a nearest-neighbour coupling of about $J = 2\pi$ 1.2kHz.

In summary, we have presented a modification of previous designs for ion quantum computation with magnetic field gradients but where the quantum information is now manipulated and stored in qutrits. A magnetic field gradient allows for individual qutrit addressing and introduces a qutrit-qutrit coupling for quantum logic. The scheme requires no additional physical resources.

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α_1	-0.5628
α_2	-0.2604
α_3	-1.9045
α_4	-2.4299
α_5	-16.5854
α_6	19.1630
α_7	-0.2738
α_8	5.3918
α_9	0.3045

TABLE I: The angles α_i written in multiples of π required in order to execute the pulse sequence for the phase gate for qutrits given in (17).

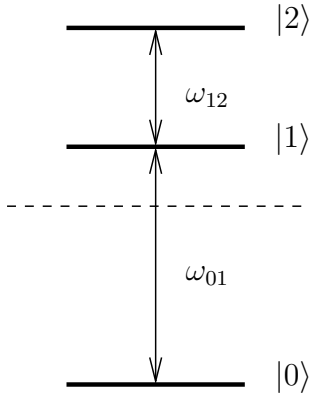


FIG. 1: The qutrit levels $\{|0\rangle, |1\rangle, |2\rangle\}$ under consideration with ω_{12} and ω_{01} the resonant frequencies for the $|1\rangle \leftrightarrow |2\rangle$ and $|0\rangle \leftrightarrow |1\rangle$ transitions respectively.

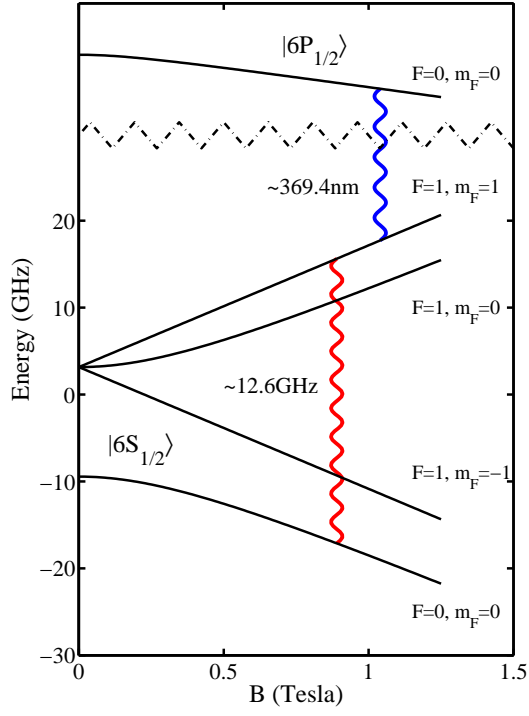


FIG. 2: The hyperfine Zeeman levels of an $^{171}\text{Yb}^+$ ion in a spatially varying magnetic field.